# CBSE SAMPLE PAPER - 07

## Class 12 - Mathematics

**Time Allowed: 3 hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

**Section A**

1. $\int \sqrt{\frac{x}{1-x}} dx$ is equal to **[1]**

a) $\sin^{-1}|\sqrt{x} - \sqrt{x(1-x)}| + c$

b) $\lim^{-1}(\sqrt{x(1-x)}) + C$

c) $\sin^{-1}\sqrt{x} - \sqrt{x(1-x)} + c$

d) $\sin^{-1}\sqrt{x} + c$

2. The Cartesian equations of a line are $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$. What is its vector equation? **[1]**

a) none of these

b) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

c) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2i + 3j - 2k)$

d) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k})$

3. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of is: **[1]**

a) $\frac{5\pi}{12}$

b) $\frac{2\pi}{3}$

c) $\frac{5\pi}{6}$

d) $\frac{\pi}{4}$

4. For two mutually exclusive events A and B, P(A) = 0.2 and P($\bar{A} \cap B$) = 0.3. What is P(A|(A ∪ B)) equal to? **[1]**

a) $\frac{2}{7}$

b) $\frac{2}{5}$

c) $\frac{2}{3}$

d) $\frac{1}{2}$

5. $\int \frac{dx}{e^x + e^{-x}}$ is equal to **[1]**

a) log $(e^x + e^{-x})$ + C

b) log $(e^x - e^{-x})$ + C

c) $\tan^{-1}(e^{-x})$ + C

d) $\tan^{-1}(e^{x})$ + C

6. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then, the events A and B are **[1]**

a) independent and equally likely

b) independent but not equally likely

c) equally likely but not independent

d) mutually exclusive and independent

7. The area of the region { ( x , y ) : $x^2 + y^2 \leqslant 1 \leqslant x + y$ } is equal to **[1]**

a) $\frac{\pi-2}{4}$ sq. units

b) $\frac{3\pi-2}{4}$ sq. units

c) none of these

d) $\frac{1}{2}$ sq. units

8. The lines $l_1$ and $l_2$ intersect. The shortest distance between them is **[1]**

a) infinity

b) negative

c) positive

d) zero

9. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to **[1]**

a) $= \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

b) $3\hat{i} - 9\hat{j} - 5\hat{k}$

c) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

d) $(-3\hat{i} + 9\hat{j} + 5\hat{k})$

10. The general solution of the differential equation $\frac{dy}{dx}$ + y cot x = cosec x, is **[1]**

a) x + y cos x = C

b) y + x (sin x + cos x) = C

c) x + y sin x = C

d) y sin x = x + C

11. The area (in sq units) of the region bounded by the parabola, y = $x^2$ + 2 and the lines, y = x + 1, x = 0 and x = 3, is **[1]**

a) $\frac{21}{2}$

b) $\frac{17}{4}$

c) $\frac{15}{4}$

d) $\frac{15}{2}$

12. $\int \frac{dx}{(x+1)\sqrt{4x+3}}$ is equal to **[1]**

a) $4\tan^{-1}\sqrt{4x+3}$ + c

b) $\tan^{-1}\sqrt{4x+3}$ + c

c) $3\tan^{-1}\sqrt{4x+3}$ + c

d) $2\tan^{-1}\sqrt{4x+3}$ + c

13. The function f(x ) = $x^3$ - 27x + 8 is increasing when **[1]**

a) $|x| < 3$

b) -3 < x < 3

c) None of these

d) $|x|$ > 3

14. If A = $\frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$, $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$, then A - B is equal to **[1]**

a) 2I

b) I

c) O

d) $\frac{1}{2}$I

15. If A and B are any $2 \times 2$ matrices, then det. (A+B) = 0 implies **[1]**

a) detA + det B = 0

b) det A = 0 or det B = 0

c) None of these

d) det A = 0 and det B = 0

16. Which of the following is not correct? **[1]**

a) |kA| = $k^3$|A|, where A = $[a_{ij}]_{3\times3}$

b) If A is a skew-symmetric matrix of odd order, then |A| = 0

c) $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$

d) |A| = $|A^T|$, where A = $[a_{ij}]_{3\times3}$

17. The principal value of $\sin^{-1}(\sin \frac{3\pi}{4}) = \ldots\ldots$ **[1]**

a) $\frac{\pi}{4}$

b) $\frac{3\pi}{4}$

c) $\frac{5\pi}{4}$

d) $\frac{-\pi}{4}$

18. The general solution of a differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is ? **[1]**

a) $xe^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right) dy + C$

b) $ye^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right) dy + C$

c) $y \cdot e^{\int_P dx} = \int\left(Q_1 e^{\int P_1 dx}\right) dx + C$

d) $xe^{\int P^1 dx} = \int\left(Q_1 e^{\int P_1 dx}\right) dx + C$

19. **Assertion (A):** If manufacturer can sell x items at a price of ₹$(5 - \frac{x}{100})$ each. The cost price of x items is ₹ $(\frac{x}{5} + 500)$. Then, the number of items he should sell to earn maximum profit is 240 items. **[1]**

**Reason (R):** The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100}$ - 300.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The matrix A = $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular. **[1]**

**Reason (R):** A square matrix A is said to be singular, if |A| - 0.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

## Section B

21. $\tan^{-1}(-1)$ **[2]**

22. If y = $x^3 \log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$ **[2]**

23. Find minors and cofactors of the elements of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ Verify that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$. **[2]**

OR

Using matrix method, solve the system of equations

4x + 3y + 2z = 60;

x + 2y + 3z = 45;

6x + 2y + 3z = 70.

24. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$ **[2]**

25. From a pack of 52 cards, two are drawn one by one without replacement. Find the probability that both of them are kings. **[2]**

## Section C

26. Evaluate $\int_0^{\pi/2} x^2 \sin x dx$. **[3]**

27. Show that the differential equation representing one parameter family of curves is: **[3]**

$(x^2 - y^2) = c(x^2 + y^2)^2$ is $(x^3 - 3xy^2)\, dx = (y^3 - 3x^2y)dy$

OR

Solve the following differential equation.

$\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x dy = 0$

28. If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, find the angles which the vector $2\vec{a} + \vec{b} + 2\vec{c}$ makes with the vectors $\vec{a} + \vec{b}$ and $\vec{c}$. **[3]**

OR

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors:

$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

29. Prove that: $\int\limits_0^{\pi/2} \frac{\cos^2 x}{(\sin x + \cos x)} dx = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$ **[3]**

OR

Evaluate $\int e^{-3x} \cos^3 x dx$

30. If $y = x^{\cos x} + (\cos x)^x$, prove that **[3]**

$\frac{dy}{dx} = x^{\cos x} \cdot \left\{\frac{\cos x}{x} - (\sin x) \log x\right\} + (\cos x)^x [(\log \cos x) - x \tan x]$.

31. Find the area of region bounded by the points y = -1, y = 2, $x = y^3$ and x = 0. **[3]**

**Section D**

32. Solve the following LPP graphically: **[5]**

Minimize and Maximize Z = 5x + 2y

Subject to

$-2x - 3y \leq -6$

$x - 2y \leq 2$

$3x + 2y \leq 12$

$-3x + 2y \leq 3$

$x, y \geq 0$

33. Let A and B be two sets. Show that f: $A \times B \rightarrow B \times A$ such that f(a, b) = (b, a) is **[5]**

(i) injective

(ii) bijective

OR

Let R be relation defined on the set of natural number N as follows:

R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

34. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular. **[5]**

OR

Find the image of the point with position vector $3\hat{i} + \hat{j} + 2\hat{k}$ in the plan $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$ Also, find the position vectors of the foot of the perpendicular and the equation of the perpendicular line through $3\hat{i} + \hat{j} + 2\hat{k}$.

35. Show that the function defined as follows, is continuous at x = 2 but not differentiable there at x = 2. **[5]**

$f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$

**Section E**

36. **Read the text carefully and answer the questions:** **[4]**

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight.

(ii) What is the number of days it will take for the plant to grow to the maximum height?

(iii) Verify that height of the plant is maximum after four days by second derivative test and find the maximum height of plant.

**OR**

What will be the height of the plant after 2 days?

37. **Read the text carefully and answer the questions:** **[4]**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association). Let A be a $1 \times 2$ matrix and B be a $2 \times 1$ matrix, representing the investment and interest rate on each bond respectively.


(i) Represent the given information in matrix algebra.

(ii) If ₹15000 is invested in bond X, then find total amount of interest received on both bonds?

(iii) If the trust fund obtains an annual total interest of ₹ 3200, then find the investment in two bonds.

**OR**

If the amount of interest given to old age home is ₹500, then find the amount of investment in bond Y.

38. **Read the text carefully and answer the questions:** **[4]**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

**Solution**

**CBSE SAMPLE PAPER - 07**

**Class 12 - Mathematics**

**Section A**

1. **(c)** $\sin^{-1}\sqrt{x}-\sqrt{x(1-x)}+c$

   **Explanation:** $I=\int\sqrt{\frac{x}{1-x}}dx$

   $I=\int\sqrt{\frac{x}{1-x}\times\frac{x}{x}}dx$

   $I=\int\frac{xdx}{\sqrt{x-x^2}}$

   consider,

   $x=A\frac{d(x-x^2)}{dx}+B$

   $x=A(1-2x)+B$

   $x=-2Ax+A+B$

   $-2\mathrm{A}=1\Rightarrow\mathrm{A}=\frac{-1}{2}$

   $\Rightarrow A+B=0\Rightarrow B=\frac{1}{2}$

   $I=\int\frac{\frac{-1}{2}(1-2x)+\frac{1}{2}}{\sqrt{x-x^2}}dx$

   $I=\int\left(\frac{-1}{2}\frac{1-2x}{\sqrt{x-x^2}}+\frac{1}{2\sqrt{x-x^2}}\right)dx$

   $I=\frac{-1}{2}\times2\sqrt{x-x^2}+\frac{1}{2}\int\frac{1}{\sqrt{x-x^2}}dx$

   Second term after completing square method you will get as

   $I=-\sqrt{x-x^2}+\sin^{-1}\sqrt{x}+c$

2. **(c)** $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda(2i+3j-2k)$

   **Explanation:** Fixed point is $2\hat{\imath}-\hat{\jmath}+3\hat{k}$ and the vector is $2\hat{\imath}+3\hat{\jmath}-2\hat{k}$

   Equation $(2\hat{\imath}-\hat{\jmath}+3\hat{k})+\lambda(2\hat{\imath}+3\hat{\jmath}-2\hat{k})$

3. **(b)** $\frac{2\pi}{3}$

   **Explanation:** Let cos $\alpha$, cos $\beta$, cos $\gamma$ be direction cosines of a

   $\therefore\cos\alpha=\cos\frac{\pi}{3},\cos\beta=\cos\frac{\pi}{4}$ and $\cos\gamma=\cos\theta$

   $\Rightarrow\cos^2\frac{\pi}{3}+\cos^2\frac{\pi}{4}+\cos^2\theta=1\Rightarrow\cos^2\theta=\frac{1}{4}$

   $\Rightarrow\cos\theta=\pm\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

4. **(b)** $\frac{2}{5}$

   **Explanation:** As, $\bar{A}\cap B=B-A\cap B$

   So, in the given case, $P(\bar{A}\cap B)=P(B)=0.3$ [$\because$ A and B are mutually exclusive, so $A\cap B=\phi\Rightarrow P(A\cap B)=0$]

   and $P(A|(A\cup B))=\frac{P[A\cap(A\cup B)]}{P(A\cup B)}$

   $=\frac{P(A)}{P(A)+P(B)}=\frac{0.2}{0.2+0.3}=\frac{2}{5}$

5. **(d)** $\tan^{-1}(e^x)+C$

   **Explanation:** Given Integral is: $\int\frac{dx}{e^x+e^{-x}}$

   Let $I=\int\frac{dx}{e^x+e^{-x}}$

   $=\int\frac{dx}{e^{-x}(e^{2x}+1)}$

   $=\int\frac{e^xdx}{(e^{2x}+1)}$

   Put $e^x=t\Rightarrow e^x\,dx=dt$

   $\Rightarrow\int\frac{e^xdx}{(e^{2x}+1)}=\int\frac{dt}{(t^2+1)}$

   $=\tan^{-1}t+C$

   $=\tan^{-1}(e^x)+C$

6. **(b)** independent but not equally likely

**Explanation:** Given $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}, P(\bar{A}) = \frac{1}{4}$

$\therefore \quad P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - \frac{1}{6} = \frac{5}{6}$

and $P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore$ P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$\Rightarrow \quad P(B) = \frac{1}{3} \Rightarrow$ A and B are not equally likely

$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4}$

So, events are independent.

7. **(a)** $\frac{\pi-2}{4}$ sq. units

**Explanation:** $x^2 + y^2 = 1, x + y = 1$

Meets when

$x^2 + (1 - x)^2 = 1$

$\Rightarrow x^2 + 1 + x^2 - 2x = 1$

$\Rightarrow 2x^2 - 2x = 0 \Rightarrow 2x(x - 1) = 0$

i.e. points ( 1 ,0 ) ,( 0 ,1 ). Therefore , required area is ;

$\int_0^1 \left(\sqrt{1 - x^2} - (1 - x)\right) dx$

$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$

8. **(d)** zero

**Explanation:** Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

9. **(a)** $= \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

**Explanation:** Given vectors $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$ such that $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\alpha$

So, $\vec{\beta}_1 = \lambda\alpha = \lambda(3\hat{i} + \hat{j})$

Now, $\vec{\beta}_2 = \vec{\beta}_1 - \vec{\beta} = \lambda(3\hat{i} + \hat{j}) - (2\hat{i} - \hat{j} + 3\hat{k})$

$= (3\lambda - 2)\hat{\mathbf{i}} + (\lambda + 1)\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\because \vec{\beta}_2$ is perpendicular to $\alpha$, so $\vec{\beta}_2 . \alpha = 0$

[since if non-zero vectors a and b are perpendicular to each other, then $\vec{a}.\vec{b} = 0$]

$\therefore (3\lambda - 2)(3) + (\lambda + 1)(1) = 0$

$\Rightarrow 9\lambda - 6 + \lambda + 1 = 0$

$\Rightarrow 10\lambda = 5 \Rightarrow \lambda = \frac{1}{2}$

So, $\vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$

and $\vec{\beta}_2 = \left(\frac{3}{2} - 2\right)\hat{i} + \left(\frac{1}{2} + 1\right)\hat{j} - 3\hat{k} = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

$$\therefore \vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{3}{2} & -3 \\ \frac{-1}{2} & \frac{3}{2} & -3 \end{vmatrix} = \hat{i}\left(-\frac{3}{2} - 0\right) - \hat{j}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$$

$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k} = \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$.

10. **(d)** y sin x = x + C

**Explanation:** We have,

$\frac{dy}{dx}$ + y cot x = cosec x

Comparing with $\frac{dy}{dx}$ + Py = Q of the above equition then,we get

$\Rightarrow$ P = cot x, Q = cosec x

I.F. = $e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x}$ = sin x

Multiplying on both sides by sin x

sin x $\frac{dy}{dx}$ + y cos x = 1

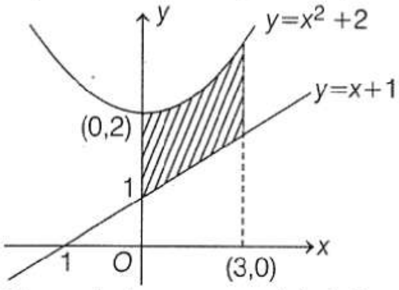
$\Rightarrow \frac{d}{dx}(y\sin x) = 1$

$\Rightarrow$ y sin x = $\int 1dx$

$\Rightarrow$ y sin x = x + C

11. **(d)** $\frac{15}{2}$

**Explanation:** Given equation of parabola is y = $x^2$ + 2 and the lines is y = x + 1




The required area = area of shaded region

$= \int_0^3 \left((x^2+2)-(x+1)\right)dx = \int_0^3 \left(x^2-x+1\right)dx$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3 = \left(\frac{27}{3} - \frac{9}{2} + 3\right) - 0$

$= 9 - \frac{9}{2} + 3 = 12 - \frac{9}{2} = \frac{15}{2}$ sq units

12. **(d)** $2\tan^{-1}\sqrt{4x+3}$ + c

**Explanation:** Let 4x + 3 = $t^2$ $\Rightarrow$ 4dx = 2t dt

$\int \frac{dx}{(x+1)\sqrt{4x+3}} = \frac{1}{2}\int \frac{t\,dt}{\left(\frac{t^2-3}{4}+1\right)t} = 2\int \frac{dt}{1+t^2}$

= $2\tan^{-1}$ t + c

= $2\tan^{-1}\sqrt{4x+3} + c$

13. **(d)** $|x| > 3$

**Explanation:** Given function

f(x) = $x^3$ - 27 + 8

f'(x) = $3x^2$ - 27x = 0

f'(x) = 3($x^2$ - 9) = 0

f'(x) = 3(x - 3) (x + 3) = 0

x = 3 or x = -3

for x > 3 f(x) is increasing

for x < -3 f(x) is increasing

$\therefore$ for $|x| > 3$ f(x) is increasing

14. **(d)** $\frac{1}{2}$I

**Explanation:** In the given question, B = $\begin{bmatrix} -\frac{1}{\pi}\cos^{-1}x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & -\frac{1}{\pi}\tan^{-1}\pi x \end{bmatrix}$

and A = $\begin{bmatrix} \frac{1}{\pi}\sin^{-1}x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & \frac{1}{\pi}\cot^{-1}\pi x \end{bmatrix}$

$\therefore$ A - B = $\begin{bmatrix} \frac{1}{\pi}(\sin^{-1}x\pi + \cos^{-1}x\pi) & 0 \\ 0 & \frac{1}{\pi}(\cot^{-1}\pi x + \tan^{-1}\pi x) \end{bmatrix}$

= $\begin{bmatrix} \frac{1}{\pi}\cdot\frac{\pi}{2} & 0 \\ 0 & \frac{1}{\pi}\cdot\frac{\pi}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}I$

15. **(c)** None of these

**Explanation:** If det (A+B) = 0 implies that A+B a Singular matrix.

16. **(c)** $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$

**Explanation:** For adding the determinants, we need to find the value of the determinants and add them.We cannot apply the method applicable for matrix addition.

17. **(a)** $\frac{\pi}{4}$

**Explanation:** $\sin^{-1}(\sin\frac{3\pi}{4}\ ) = \sin^{-1}(\sin\left(\pi - \frac{\pi}{4}\right)$

$= \sin^{-1}(\sin\frac{\pi}{4}) = \frac{\pi}{4}$ .

18. **(a)** $xe^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right)dy + C$

**Explanation:** The integrating factor of the given differential equation

$\frac{dx}{dy} + P_1 x = Q_1$ is $e^{\int P_1 dy}$

Thus, the general solution of the differential equation is given by,

$x(I.F.) = \int(Q_1 \times I.F.)dy + C$

$\Rightarrow x.e^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right)dy + C$

19. **(c)** A is true but R is false.

**Explanation:** Let S(x) be the selling price of x items and let C(x) be the cost price of x items.

Then, we have

$S(x) = (5 - \frac{x}{100})x = 5x - \frac{x^2}{100}$

and $C(x) = \frac{x}{5} + 500$

Thus, the profit function P(x) is given by

$P(x) = S(x) - C(x) = 5x - \frac{x^2}{100} - \frac{x}{5} - 500$

i.e. $P(x) = \frac{24}{5}x - \frac{x^2}{100} - 500$

On differentiating both sides w.r.t. x, we get

$P'(x) = \frac{24}{5} - \frac{x}{50}$

Now, P'(x) = 0 gives x = 240.

Also, $P'(x) = \frac{-1}{50}$.

So, $P'(240) = \frac{-1}{50} < 0$

Thus, x = 240 is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** The determinant of the matrix $A = \begin{bmatrix}1 & 2\\4 & 8\end{bmatrix}$ is $|A| = \begin{vmatrix}1 & 2\\4 & 8\end{vmatrix}$ = 8 - 8 = 0

Hence, A is a singular matrix.

**Section B**

21. Let $\tan^{-1}(-1) = y$

$\Rightarrow \tan y = -1$

$\Rightarrow \tan y = -\tan\frac{\pi}{4}$

$\Rightarrow \tan y = \tan\left(-\frac{\pi}{4}\right)$

Since, the principal value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

22. Given that: $y = x^3 \log\left(\frac{1}{x}\right)$

To prove: $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$

Proof: We have, $y = x^3 \log\left(\frac{1}{x}\right)$, then

$\frac{dy}{dx} = x^3\left(\frac{1}{\frac{1}{x}}\right)\left(\frac{(-1)}{x^2}\right) + \log\left(\frac{1}{x}\right)(3x^2) = -x^2 + 3x^2\left[\log\left(\frac{1}{x}\right)\right]$

$\frac{d^2y}{dx^2} = -2x + 6x\left[\log\left(\frac{1}{x}\right)\right] + \left\{3x^2 \times (x) \times \left(\frac{-1}{x^2}\right)\right\}$

$\frac{d^2y}{dx^2} = -5x + 6x\left[\log\left(\frac{1}{x}\right)\right]$

Now put the values in $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2$, we get

$x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2$

$= x\left\{-5x + 6x\left[\log\left(\frac{1}{x}\right)\right]\right\} - 2\left\{-x^2 + 3x^2\left[\log\left(\frac{1}{x}\right)\right]\right\} + 3x^2$

$= -5x^2 + 6x^2 \left[\log\left(\frac{1}{x}\right)\right] + 2x^2 - 6x^2 \left[\log\left(\frac{1}{x}\right)\right] + 3x^2$

$= -5x^2 + 2x^2 + 3x^2$

$= 0$

Hence Proved

23. We have, $M_{11} = \begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} = 0 - 20 = -20; A_{11} = (-1)^{1+1}(-20) = -20$

$M_{12} = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = -42 - 4 = -46; \quad A_{12} = (-1)^{1+2}(-46) = 46$

$M_{13} = \begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix} = 30 - 0 = 30; \quad A_{13} = (-1)^{1+3}(30) = 30$

$M_{21} = \begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} = 21 - 25 = -4; \quad A_{21} = (-1)^{2+1}(-4) = 4$

$M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix} = -14 - 5 = -19; \quad A_{22} = (-1)^{2+2}(-19) = -19$

$M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13; \quad A_{23} = (-1)^{2+3}(13) = -13$

$\mathbf{M}_{31} = \begin{vmatrix} -3 & 5 \\ 0 & 4 \end{vmatrix} = -12 - 0 = -12; \quad A_{31} = (-1)^{3+1}(-12) = -12$

$M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = 8 - 30 = -22; \quad A_{32} = (-1)^{3+2}(-22) = 22$

and $M_{33} = \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 0 + 18 = 18; \quad A_{33} = (-1)^{3+3}(18) = 18$

Now $a_{11} = 2$, $a_{12} = -3$, $a_{13} = 5$; $A_{31} = -12$, $A_{32} = 22$, $A_{33} = 18$

So, $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$

$= 2(-12) + (-3)(22) + 5(18) = -24 - 66 + 90 = 0$

OR

Given set of equations are:-

$4x + 3y + 2z = 60$

$x + 2y + 3z = 45$

$6x + 2y + 3z = 70$

Converting the following set of equations in matrix form, we have

$AX = B$

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$R_2 \to 4R_2 - R_1$

$R_3 \to 2R_3 - 3R_1$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Again converting into equations, we get

$4x + 3y + 2z = 60$

$5y + 10z = 120$

$-5y = -40$

$Y = 8$

$5 \times 8 + 10z = 120$

$10z = 120 - 40$

$10z = 80$

$z = 8$

$4x + 3 \times 8 + 2 \times 8 = 60$

$4x = 60 - 24 - 16$

$4x = 20$

$x = 5$

$\therefore x = 5, y = 8, z = 8$

24. L.H.S $= \left(\vec{a}-\vec{b}\right) \times \left(\vec{a}+\vec{b}\right)$

$= \vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$

$= 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-0\ [as\,\vec{a}\times\vec{b}=-\vec{b}\times\ \vec{a}]$

$= 2(\vec{a}\times\vec{b})$

25. Total number of all favorable cases is n(S) = 52

Let A be the event that first card drawn is a king. There are four kings in the pack. Hence, the probability of the first card is a king is P(A) = $\frac{4}{52}$

Let B be the event of getting King in the second draw. When the card drawn in the first draw is a king, then there are 3 kings left in the pack as the cards are not replaced. Therefore, the probability of the second card is also king is

$P(B|A)=\frac{3}{51}$

Then the probability of getting two kings without replacement is given by

$P(A\cap B)$

$= \mathrm{P(A)P(B|A)}$

$\Rightarrow \frac{4}{52}\times\frac{3}{51}$

$= \frac{1}{13}\times\frac{1}{17}=\frac{1}{221}$

The probability that both of them are kings is $\frac{1}{221}$

**Section C**

26. Given, $I=\int_0^{\pi/2} x^2\sin x dx$

$I=\int_0^{\pi/2} \underset{I}{x^2}\sin \underset{II}{x}\, dx$

By using integration by parts we get ,

$I=[-x^2\cos x]_0^{\pi/2}+2\int_0^{\pi/2}\underset{I}{x}\cos\underset{II}{x}\, dx$

$=[-x^2\cos x]_0^{\pi/2}+2\left[[x(\sin x)]_0^{\pi/2}-\int_0^{\pi/2}1\cdot(\sin x)dx\right]$

$=[-x^2\cos x]_0^{\pi/2}+2\left[[x(\sin x)]_0^{\pi/2}+[cosx]_0^{\pi/2}\right]$

$\therefore I=\int_0^{\pi/2}x^2\sin xdx=\left[-x^2\cos x+2(x\sin x+\cos x)\right]_0^{\pi/2}$

$=\left[-\left(\frac{\pi}{2}\right)^2\cos\left(\frac{\pi}{2}\right)+2\left(\frac{\pi}{2}\sin\frac{\pi}{2}+\cos\frac{\pi}{2}\right)-2(0+\cos 0)\right]$

$=-\frac{\pi^2}{4}\times 0+2\left(\frac{\pi}{2}+0\right)-2(0+1)$

$\therefore I=\pi-2$

27. The given equation of one parameter family of curves is

$(x^2 - y^2) = c(x^2 + y^2)^2$ ...(i)

Differentiating (i) with respect to x, have,

$2x-2y\frac{dy}{dx}=2c\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)$

$\Rightarrow\quad \left(x-y\frac{dy}{dx}\right)=2c\left(x^2+y^2\right)\left(x+y\frac{dy}{dx}\right)$ ...(ii)

On using the value of c obtained from (i) in (ii), we get

$\left(x-y\frac{dy}{dx}\right)=\frac{2\left(x^2-y^2\right)\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2}\left(x+y\frac{dy}{dx}\right)$

$\Rightarrow\quad \left(x^2+y^2\right)\left(x-y\frac{dy}{dx}\right)=2\left(x^2-y^2\right)\left(x+y\frac{dy}{dx}\right)$

$\Rightarrow\quad \left\{x\left(x^2+y^2\right)-2x\left(x^2-y^2\right)\right\}=\frac{dy}{dx}\left\{2y\left(x^2-y^2\right)+y\left(x^2+y^2\right)\right\}$

$\Rightarrow\quad \left(3xy^2-x^3\right)=\frac{dy}{dx}\left(3x^2y-y^3\right)$

$\Rightarrow$ $(x^3 - 3xy^2)dx = (y^3 - 3x^2y\,)dy$, which is the given differential equation.

OR

According to given question ,

Given differential equation is,

$\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+xdy=0$

$\Rightarrow\quad \frac{dy}{dx}=\frac{y}{x}-\sin^2\left(\frac{y}{x}\right)$

which is a homogeneous differential equation as $\frac{dy}{dx}=F\left(\frac{y}{x}\right)$.

Put $y=vx$,

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = v - \sin^2 v$

$\Rightarrow \quad cosec^2 v dv = -\frac{dx}{x}$

On integrating both sides, we get

$\int cosec^2 v dv + \int \frac{dx}{x} = 0$

$\Rightarrow \quad -\cot v + \log|x| = C$

$\Rightarrow \quad -\cot\left(\frac{y}{x}\right) + \log|x| = C \quad \left[\because v = \frac{y}{x}\right]$

$\Rightarrow y = x \cdot \cot^{-1}(\log x - \text{C})$

which is the required solution.

28. Here it is given that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes

Let the vector $\overrightarrow{p} = (2\overrightarrow{a} + \overrightarrow{b} + 2\overrightarrow{c})$ makes angles $\alpha, \beta, \gamma$ respectively with the vector $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$

Given that $\overrightarrow{a}| = |\overrightarrow{b}| = |\overrightarrow{c}|$ and $\vec{a} \cdot \vec{b} = \vec{c} \cdot \vec{a} = 0$

$\cos\alpha = \frac{(2\vec{a}+\vec{b}+2\vec{c})-\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}$

$= \frac{2|a|^2}{3|\vec{a}||\vec{a}|} = \frac{2}{3}$

$\Rightarrow \alpha = \cos^{-1}\frac{2}{3}$

$\cos\beta = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{b}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{b}|} = \frac{|\vec{b}|^2}{3|\vec{b}||\vec{b}|} = \frac{1}{3}$

$\Rightarrow \beta = \cos^{-1}\frac{1}{3}$

$\cos\gamma = \frac{(2\overrightarrow{a}+\overrightarrow{b}+2\overrightarrow{c})\cdot\overrightarrow{c}}{|2\overrightarrow{a}+\overrightarrow{b}+2\overrightarrow{c}||\overrightarrow{c}|} = \frac{2|\overrightarrow{c}|^2}{3|\overrightarrow{c}||\overrightarrow{c}|} = \frac{2}{3}$

OR

if $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{\text{k}}$ then, $[\overrightarrow{\text{ab}}\overrightarrow{\text{c}}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1}a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Given:

$\overrightarrow{\text{a}} = 2\hat{\text{i}} + 3\hat{\text{j}}, \overrightarrow{\text{b}} = \hat{\text{i}} + 2\hat{\text{j}} - \hat{\text{k}}, \overrightarrow{\text{c}} = 3\hat{\text{i}} - \hat{\text{j}} + 2\hat{\text{k}}$

we know that the volume of parallelepiped whose three adjacent edges are

$\vec{a}, \vec{b}\vec{c}$ is equal to $|[\overrightarrow{\text{ab}}\overrightarrow{\text{c}}]|$.

we have

$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$

now, using

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1}a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

= 2(4 – 1) – 3(2 + 3) + 4( – 1 – 6)

= – 37

therefore, the volume of the parallelepiped is

$[\overrightarrow{\text{ab}}\overrightarrow{\text{c}}]$ = |-37| = 37 cubic unit....................................................

29. Let the given integral be, $y = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx$ .....(i)

Using theorem of definite integral we have,

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\frac{\pi}{2}} \frac{\cos^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx$

$y = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ .....(ii)

Adding eq.(i) and eq.(ii)

$2y = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x+\cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x} dx$

$2y = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x+\cos x} dx$

$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x} dx$

$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\sin\left(x+\frac{\pi}{4}\right)} dx$

$y = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} cosec\left(x+\frac{\pi}{4}\right) dx$

$y = \frac{1}{2\sqrt{2}} \left(\ln\left(cosec\left(x+\frac{\pi}{4}\right) - \cot\left(x+\frac{\pi}{4}\right)\right)\right)_0^{\frac{\pi}{2}}$

$y = \frac{1}{2\sqrt{2}} \left(\ln\left(cosec\frac{3\pi}{4} - \cot\frac{3\pi}{4}\right) - \ln\left(cosec\frac{\pi}{4} - \cot\frac{\pi}{4}\right)\right)$

$y = \frac{1}{2\sqrt{2}} \ln\frac{\sqrt{2}+1}{\sqrt{2}-1}$

$y = \frac{1}{2\sqrt{2}} \ln(\sqrt{2}+1)^2 = \frac{1}{\sqrt{2}} \ln(\sqrt{2}+1)$

OR

Given integral is: $\int e^{-3x} \cos^3 x dx$

Using trigonometric identity cos 3x = 4 $\cos^3$ x - 3 cos x

$\Rightarrow \int e^{-3x} \cos^3 x dx = \frac{1}{4} \int e^{-3x} (\cos 3x + 3\cos x) dx$

$\Rightarrow \frac{1}{4} \int e^{-3x} (\cos 3x + 3\cos x) dx = \frac{1}{4} \int e^{-3x} \cos 3x dx + \frac{3}{4} \int e^{-3x} \cos x dx$ ...(i)

Using a general formula i.e.

$\Rightarrow \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2}$ (a cos bx + b sin bx)

$\Rightarrow \int e^{-3x} \cos 3x dx = \frac{e^{-3x}}{(-3)^2+3^2}$ ((-3) cos 3x + 3 sin 3x)

$\Rightarrow \frac{e^{-3x}}{(-3)^2+3^2}$ ((-3) cos 3x + 3 sin 3x) = $\frac{e^{-3x}}{6}$ (sin 3x - cos 3x) .....(ii)

$\Rightarrow \int e^{-3x} \cos x dx = \frac{e^{-3x}}{(-3)^2+1^2}$ ((-3) cos 3x + 3 sin 3x) = $\frac{e^{-3x}}{10}$ (sin x - 3 cos x)

$= \frac{e^{-3x}}{10}$ (sin x - 3 cos x) .....(iii)

On putting (ii) and (iii) in (i)

$\Rightarrow \frac{1}{4} \int e^{-3x} \cos 3x dx + \frac{3}{4} \int e^{-3x} \cos x dx = \frac{e^{-3x}}{4\times 6} (\sin 3x - \cos 3x) + \frac{3e^{-3x}}{4\times 10}$ (sin x - cos x)

$\Rightarrow \int e^{-3x} \cos^3 x dx = e^{-3x} \left\{\frac{(\sin 3x - \cos 3x)}{24} + \frac{3(\sin x - 3\cos x)}{40}\right\} + C$

30. Here,it is given that y = $x^{\cos x}$ + $(\cos x)^x$, Let u = $x^{\cos x}$, and v = $(\cos x)^x$

y = u + v .....(i)

On differentiating both sides of (i) w.r.t. x,we get

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ .....(ii)

u = $x^{\cos x}$

Taking log both sides, we get log u = log ($x^{\cos x}$) $\Rightarrow$ log u = cos x log x ......(iii)

On differentiating both sides of (iii) w.r.t. x ,we get

$\frac{1}{u} \cdot \frac{du}{dx} = \cos x \cdot \frac{1}{x}$ + log x (-sin x)

$\Rightarrow \frac{du}{dx}$ = u($\frac{\cos x}{x}$ - log x sin x)

$\Rightarrow \frac{du}{dx}$ = $x^{\cos x}$($\frac{\cos x}{x}$ - log x sin x)

v = $(\cos x)^x$

Taking both sides, we get,

log v = log ($(\cos x)^x$) ......(iv)

On differentiating both sides of (iii) w.r.t. x,we get

$\frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{1}{\cos x} \cdot (-\sin x)$ + log (cos x)]

$\Rightarrow \frac{dv}{dx}$ = $(\cos x)^x$ [log (cos x) - x tan x]

Substituting the value of $\frac{du}{dx}$ and $\frac{dv}{dx}$ in equation (ii),

$\frac{dy}{dx}$ = $x^{\cos x}$($\frac{\cos x}{x}$ - log x sin x) + $(\cos x)^x$ [log (cos x) - x tan x]

31. A rough sketch of the curve x = y is shown in the below Figure

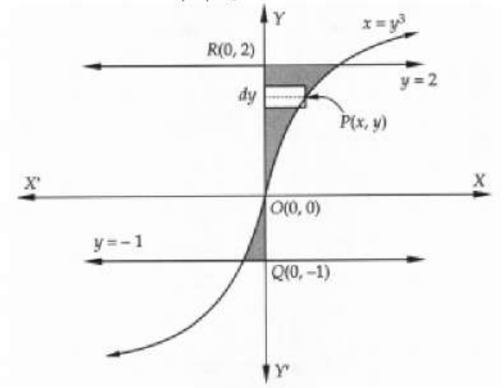
Clearly, y = - 1 and y = 2 are the straight lines parallel to x-axis.

Therefore the required region is shaded in Figure.

When we slice this region into horizontal strips, we observe that each strip has its one end on y-axis and other end on the curve x = $y^3$.

So, the approximating rectangle are shown in Fig. has, length = $|x|$ and Width = dy

therefore Area = $|x|$ dy.




Since the approximating rectangle can move vertically from y = - 1 to y = 2. So, required area denoted by A is given by

$A=\int_{-1}^{2}|x|dy$

$\Rightarrow A=\int_{-1}^{0}|x|dy+\int_{0}^{2}|x|dy$

$\Rightarrow A=\int_{-1}^{0}-xdy+\int_{0}^{2}xdy$ [$\because x<0$ for $-1\leq y<0$ and $x>0$ for $0\leq y\leq 2\}$

$\Rightarrow A=\int_{1}^{0}-y^3dy+\int_{0}^{2}y^3dy$ [$\because$ P(x, y) lies on x = $y^3$ $\therefore$ x = $y^3$]

$\Rightarrow A=-\left[\frac{y^4}{4}\right]_{-1}^{0}+\left[\frac{y^4}{4}\right]_{0}^{2}=\frac{1}{4}+4=\frac{17}{4}$ sq. units

**Section D**

32. Converting the given inequations into equations, we get

2x + 3y = 6, x - 2y = 2, 3x + 2y = 12, - 3x + 2y = 3, x = 0 and y = 0

Region represented by - 2x - 3y $\leq$ - 6:

The line - 2x - 3y = - 6 or, 2x + 3y = 6 cuts OX and OY at $A_1$ (3, 0) and $B_1$ (0, 2) respectively. Join these points to obtain the line 2x + 3y - 6 = 0.

Since O (0, 0) does not satisfy the inequation - 2x - 3y $\leq$ - 6.

So, the region represented by - 2x - 3y $\leq$ - 6 is that part of XOY-plane which does not contain the origin.

Region represented by x - 2y $\leq$ 2:

The line x - 2y = 2 meets the coordinate axes at $A_2$ (2, 0) and $B_2$ (0, -1 ).

Join these points to obtain x - 2y = 2. Since (0,0) satisfies the inequation x - 2y $\leq$ 2, so the region containing the origin represents the solution set of this inequation.

Region represented by 3x + 2y $\leq$ 12:

The line 3x + 2y $\leq$ 12 intersects OX and OY at $A_3$ (4, 0) and $B_3$ (0, 6).

Join these points to obtain the line 3x + 2y = 12. Clearly, (0, 0) satisfies the inequation 3x + 2y $\leq$ 12.

So, the region containing the origin is the solution set of the given inequations.

Region represented by - 3x + 2y $\leq$ 3:

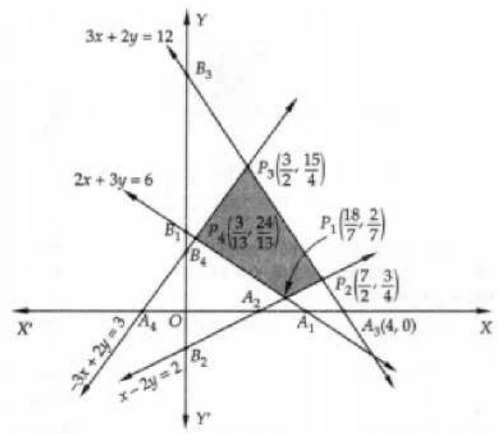
The line - 3x + 2y = 3 intersects OX and OY at $A_4$ (- 1, 0) and $B_4$ (0, 3/2). Join these points to obtain the line - 3x + 2y = 3.

Clearly, (0, 0) satisfies this inequation. So, the region containing the origin represents the solution set of the given inequation.

Region represented by x $\geq$ 0, y $\geq$ 0:

Clearly, XOY quadrant represents the solution set of these two inequations.

The shaded region shown in a figure represents the common solution set of the above inequations. This region is the feasible region of the given LPP.




The coordinates of the corner-points (vertices) of the shaded feasible region $P_1$ $P_2$ $P_3$ $P_4$ are $P_1(\frac{18}{7}, \frac{2}{7})$, $P_2(\frac{7}{2}, \frac{3}{4})$, $P_3(\frac{3}{2}, \frac{15}{4})$ and $P_4(\frac{3}{13}, \frac{24}{13})$.

These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

The values of the objective function at these points are given in the following table:

| Point (x, y) | Value of the objective function Z = 5x + 2y |
|---|---|
| $P_1(\frac{18}{7}, \frac{2}{7})$ | $Z = 5 \times \frac{18}{7} + 2 \times \frac{2}{7} = \frac{94}{7}$ |
| $P_2(\frac{7}{2}, \frac{3}{4})$ | $Z = 5 \times \frac{7}{2} + 2 \times \frac{3}{4} = 19$ |
| $P_3(\frac{3}{2}, \frac{15}{4})$ | $Z = 5 \times \frac{3}{2} + 2 \times \frac{15}{4} = 15$ |
| $P_4(\frac{3}{13}, \frac{24}{13})$ | $Z = 5 \times \frac{3}{13} + 2 \times \frac{24}{13} = \frac{63}{13}$ |

Clealy, Z is minimum at $x = \frac{3}{13}$ and $y = \frac{24}{13}$ and maximum at $x = \frac{7}{2}$ and $y = \frac{3}{4}$. The minimum and maximum values of Z are $\frac{63}{13}$ and 19 respectively.

33. i. Let $(a_1\ b_1)$ and $(a_2, b_2) \in A \times B$ such that

$f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\Rightarrow a_1 = a_2$ and $b_1 = b_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

Therefore, f is injective.

ii. Let (b, a) be an arbitrary

Element of $B \times A$. then $b \in B$ and $a \in A$

$\Rightarrow (a, b)\ ) \in (A \times B)$

Thus for all $(b, a) \in B \times A$ their exists $(a, b)\ ) \in (A \times B)$

such that

f (a, b) = (b, a)

So $f: A \times B \rightarrow B \times A$

is an onto function.

Hence f is bijective.

OR

Given that,

R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}

Domain = {1,2,3,.......,20}

Range = {1,3,5,7......,39}

R is not reflexive as $(2, 2) \notin R$ as

$2 \times 2 + 2 \neq 41$

R is not symmetric

as $(1, 39) \in R$ but $(39, 1) \notin R$

R is not transitive

as $(11, 19) \in R$, $(19, 3) \in R$

But (11, 3) $\notin$ R

Hence, R is neither reflexive, nor symmetric and nor transitive.

34. Suppose the point (1, 0, 0) be P and the point through which the line passes be Q(1,-1,-10). The line is parallel to the vector $\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$

Now,

$\overrightarrow{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$

$\therefore \vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$

$= 38\hat{i} + 20\hat{j} - 2\hat{k}$

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{38^2 + 20^2 + 2^2}$

$= \sqrt{1444 + 400 + 4}$

$= \sqrt{1848}$

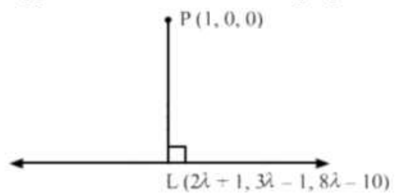
d = $\frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

$= \frac{\sqrt{1848}}{\sqrt{77}}$

$= \sqrt{24}$

$= 2\sqrt{6}$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-




The coordinates of a general point on the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are given by

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$

$\Rightarrow x = 2\lambda + 1$

$y = -3\lambda - 1$

$z = 8\lambda - 10$

Suppose the coordinates of L be

$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

Since,The direction ratios of PL are proportional to,

$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0$, i.e., $2\lambda, -3\lambda - 1, 8\lambda - 10$

Since,The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10) = 0$

$\Rightarrow \lambda = 1$ Substituting $\lambda = 1$ in $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$

$= \frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$

$\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})$

OR

Let P be the point with position vector $\vec{p} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and M be the image of P in the plane $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

In addition, let Q be the foot of the perpendicular from P on to the given plane. So, Q is the midpoint of PM.

Direction ratios of PM are proportional to 2, –1, 1 as PM is normal to the plane and parallel to $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

Recall the vector equation of the line passing through the point with position vector$\vec{r}$ and parallel to vector $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$

Here,$\vec{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

Hence, the equation of PM is

$\vec{r} = (3\hat{i} + \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\therefore \vec{r} = (3+2\lambda)\hat{i} + (1-\lambda)\hat{j} + (2+\lambda)\hat{k}$

Let the position vector of M be $\vec{m}$. As M is a point on this line, for some scalar α, we have

$\Rightarrow \vec{m} = (3+2\alpha)\hat{i} + (1-\alpha)\hat{j} + (2+\alpha)\hat{k}$

Now, let us find the position vector of Q, the midpoint of PM.

Let this be $\vec{q}$.

Using the midpoint formula, we have

$\vec{q} = \frac{\vec{p}+\vec{m}}{2}$

$\Rightarrow \vec{q} = \frac{[3\hat{i}+\hat{j}+2\hat{k}]+[(3+2\alpha)\hat{i}+(1-\alpha)\hat{j}+(2+\alpha)\hat{k}]}{2}$

$\Rightarrow \vec{q} = \frac{(3+(3+2\alpha))\hat{i}+(1+(1-\alpha))\hat{j}+(2+(2+\alpha))\hat{k}}{2}$

$\Rightarrow \vec{q} = \frac{(6+2\alpha)\hat{i}+(2-\alpha)\hat{j}+(4+\alpha)\hat{k}}{2}$

$\therefore \vec{q} = (3+\alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}$

This point lies on the given plane, which means this point satisfies the plane equation $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

$\Rightarrow \left[(3+\alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}\right] \cdot (2\hat{\imath} - \hat{j} + \hat{k}) = 4$

$\Rightarrow 2(3+\alpha) - \left(\frac{2-\alpha}{2}\right)(1) + \left(\frac{4+\alpha}{2}\right)(1) = 4$

$\Rightarrow 6 + 2\alpha + \frac{4+\alpha-(2-\alpha)}{2} = 4$

$\Rightarrow 2\alpha + (1+\alpha) = -2$

$\Rightarrow 3\alpha = -3$

$\therefore \alpha = -1$

We have the image $\vec{m} = (3+2\alpha)\hat{i} + (1-\alpha)\hat{j} + (2+\alpha)\hat{k}$

$\Rightarrow \vec{m} = [3+2(-1)]\hat{i} + [1-(-1)]\hat{j} + [2+(-1)]\hat{k}$

$\therefore \vec{m} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

Therefore, the image is (1, 2, 1)

The foot of the perpendicular $\vec{q} = (3+\alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}$

$\Rightarrow \vec{q} = [3+(-1)]\hat{i} + \left[\frac{2-(-1)}{2}\right]\hat{j} + \left[\frac{4+(-1)}{2}\right]\hat{k}$

$\therefore \vec{q} = 2\hat{i} + \frac{3}{2}\hat{j} + \frac{3}{2}\hat{k}$

Thus, the position vector of the image is $\hat{i} + 2\hat{j} + \hat{k}$ and that of the foot of the perpendicular is $2\hat{\imath} + \frac{3}{2}\hat{\jmath} + \frac{3}{2}\hat{k}$

35. According to the question, we have to show that the function defined is continuous at x = 2 but not differentiable at x = 2.

f(x) = $\begin{cases} 3x-2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x-4, & x > 2 \end{cases}$

let us check its Continuity at x = 2

LHL = $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x^2 - x)$

$\Rightarrow$LHL = $\lim_{h\to 0} [2(2-h)^2 - (2-h)]$

$= \lim_{h\to 0} [2(4 + h^2 - 4h) - (2-h)]$

$= \lim_{h\to 0} (8 + 2h^2 - 8h - 2 + h)$

$\Rightarrow$LHL = 8 - 2 = 6

and RHL = $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (5x-4)$

$\Rightarrow$ RHL = $\lim_{h\to 0} [5(2+h) - 4]$

$= \lim_{h\to 0} (10 + 5h - 4) = = \lim_{h\to 0} (5h + 6)$

$\Rightarrow$ RHL = 6

Also, f(2) = $2(2)^2 - 2 = 8 - 2 = 6$

Since, LHL = RHL = f(2)

Therefore function f(x) is continuous at x = 2.

We will now check the differentiabililty of the given function at x = 2.

Differentiability at x = 2

LHD = $\lim_{h\to 0} \frac{f(2-h)-f(2)}{-h}$

$\Rightarrow$ LHD $= \lim_{h \to 0} \frac{\left[2(2-h)^2-(2-h)\right]-[8-2]}{-h}$

$= \lim_{h \to 0} \frac{2\left(4+h^2-4h\right)-(2-h)-6}{-h}$

$= \lim_{h \to 0} \frac{8+2h^2-8h-2+h-6}{h}$

$= \lim_{h \to 0} \frac{h(2h-7)}{-h} = \lim_{h \to 0}$ -(2h - 7)

$\Rightarrow$ LHD = 7

and RHD = $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$

$= \lim_{h \to 0} \frac{[5(2+h)-4]-[8-2]}{h}$

$= \lim_{h \to 0} \frac{(6+5h)-(6)}{h} = \lim_{h \to 0} \frac{5h}{h}$

$\Rightarrow$ RHD = 5

Since, LHD $\neq$ RHD

Therefore, function f(x) is not differentiable at x = 2.

Therefore, f(x) is continuous at x = 2 but not differentiable at x=2.

## Section E

36. **Read the text carefully and answer the questions:**

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation y = 4x - $\frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

(i) The rate of growth = $\frac{dy}{dx}$

$= \frac{d(4x-\frac{1}{2}x^2)}{dx}$

= 4 - x

(ii) For the height to be maximum or minimum

$\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{d\left(4x-\frac{1}{2}x^2\right)}{dx} = 4 - \frac{1}{2} \cdot 2x = 0$

$\frac{dy}{dx}$ = 4 - x = 0

$\Rightarrow$ x = 4

$\therefore$ Number of required days = 4

(iii) $\frac{dy}{dx}$ = 4 - x

$\Rightarrow \frac{d^2y}{dx^2} = -1 < 0$

$\Rightarrow$ Function attains maximum value at x = 4

We have

y = 4x - $\frac{1}{2}x^2$

$\therefore$ when x = 4 the height of the plant will be maximum which is y = $4 \times 4 - \frac{1}{2} \times (4)^2$ = 16 - 8 = 8 cm

OR

We have, y = 4x - $\frac{1}{2}x^2$

$\therefore$ When x = 4 the height of the plant will be maximum which is

y = $4 \times 4 - \frac{1}{2} \times (4)^2$

= 8 - 2 = 6 cm

37. **Read the text carefully and answer the questions:**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association). Let A be a 1 × 2 matrix and B be a 2 × 1 matrix, representing the investment and interest rate on each bond respectively.


(i) If ₹15000 is invested in bond X, then the amount invested in bond Y = ₹(35000 - 15000) = ₹20000

Investment $A = \overset{\quad X \qquad Y}{[15000 \quad 20000]}$; $B = \begin{matrix} X \\ Y \end{matrix}\overset{\text{Invest rate}}{\begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}}$

(ii) The amount of interest received on each bond is given by

$AB = [15000\ 20000] \times \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$

$= [15000 \times 0.1 + 20000 \times 0.08] = [1500 + 1600] = 3100$

(iii) Let ₹x be invested in bond X and then ₹(35000 - x) will be invested in bond Y.

Now, total amount of interest is given by

$[x\ 35000 - x] \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} = [0.1x + (35000 - x)\ 0.08]$

But, it is given that total amount of interest = ₹3200

$\therefore 0.1x + 2800 - 0.08x = 3200$

$\Rightarrow 0.02\ x = 400 \Rightarrow x = 20000$

Thus, ₹20000 invested in bond X and ₹35000 - ₹20000

= ₹15000 invested in bond Y.

OR

Let ₹x invested in bond X, then we have

$x \times \frac{10}{100} = 500 \Rightarrow x = 5000$

Thus, the amount invested in bond X is ₹5000 and so investment in bond Y be ₹(35000 - 5000) = ₹30000

38. **Read the text carefully and answer the questions:**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

$E_1 = PQ$, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$

$P\left(\frac{E}{E_1}\right) = 0\,, P\left(\frac{E}{E_2}\right) = 0\,, P\left(\frac{E}{E_3}\right) = 1\,, P\left(\frac{E}{E_4}\right) = 1$

$P(E) = P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right) + P(E_4)\cdot P\left(\frac{E}{E_4}\right)$

= 0.14 + 0.24 + = 0.38

(ii)

By Bayes' Theorem, $P\left(\frac{E_3}{E}\right) = \dfrac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$

$= \frac{0.14}{0.38} = \frac{7}{19}$

**NOTE:** The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses $E_1$ and $E_2$ are actually eliminated as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$